FOR STUDENTS ENTERING: ALGEBRA II

St. Marys Catholic Middle School

June 2016

SMCMS Parents and Students;

Attached you will find a packet of math problems that should be completed during the summer months. <u>WE ARE ASKING THE STUDENTS TO COMPLETE THE EVEN PROBLEMS IN THIS PACKET.</u> All of the problems are math concepts that were covered during the 2015-16 school year in your/your child's math class. The purpose of this summer assignment is to help you retain the information that was learned over the past 9 months. All math classes are stepping stones for the next class so it is important that knowledge is retained from year to year.

The packet should be completed prior to the start of the 2016-17 school year. The packet information may be taken for a grade in the next school year. In order to help students, math teachers from SMCMS and ECCHS will be available during July and August to help students with any difficulty they encounter. The schedule of dates and times will be available on www.eccss.org later in June.

Please contact Mr. Schneider at schneiderj@eccss.org or 834-2665 x214 if you have any questions.

Attention parents of incoming 6th graders: If your child is participating in the Summer Math Program at St. Marys Catholic Elementary School, the may complete the summer math packet, but it will be optional. Participation in the Summer Math Program will satisfy the summer math packet requirement.

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1 Basic Concepts of Algebra

1–1 Real Numbers and Their Graphs

Objective: To graph real numbers on a number line, to compare numbers, and to find their absolute values.

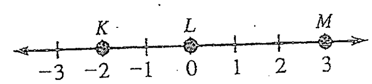
Vocabulary

Real numbers The set consisting of all the rational and irrational numbers.
A rational number is the result of dividing an integer by a nonzero integer.
An irrational number is one that is not rational.

Examples of rational numbers: $5 \quad 0 \quad -2 \quad 6.3 \quad \frac{4}{7} \quad 1.33\ldots$

Examples of irrational numbers: $\pi \quad \sqrt{2} \quad \sqrt{5}$

Coordinate of a point The real number paired with that point on a number line. Example: The coordinate of point M is 3.



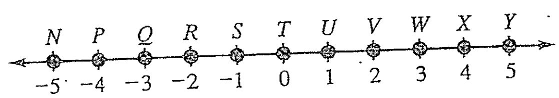
Origin The graph of zero on a number line.

Opposites On a number line, numbers that are the same distance from zero but on opposite sides of it. Example: -2 and 2

Absolute value of a number On a number line, the distance between the graph of the number and zero. Examples: The absolute value of 3 is 3 (write $|3| = 3$). The absolute value of -3 is also 3 (write $|-3| = 3$).

Symbols $-$ (opposite of) $\quad | \; |$ (absolute value) $\quad >$ (is greater than) $\quad <$ (is less than)

Example 1 Find the coordinate of the point one fourth of the way from S to W on the number line below.



Solution The distance from S to W is 4 units. One fourth of 4 is 1. Move 1 unit to the right of S to find the desired point. The coordinate is 0.

Find the coordinate of each point using the number line in Example 1.

1. Y
2. T
3. U
4. P
5. X
6. N
7. The point halfway between Q and Y
8. The point one fourth of the way from R to V
9. The point two thirds of the way from P to V
10. The point three fourths of the way from Q to W

Example 2 Write each statement using symbols.

a. Five is greater than negative two.

b. Negative ten is less than zero.

Solution a. $5 > -2$ b. $-10 < 0$

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1–2 Simplifying Expressions

Objective: To review the methods used to simplify numerical expressions and to evaluate algebraic expressions.

Vocabulary

Variable A symbol, usually a letter, used to represent one or more numbers.

Algebraic expression A numerical expression; a variable; or a sum, difference, product, or quotient that contains one or more variables.
Examples: $6 + 11$ $\quad x \quad$ $x^2 - 5y + z$

Simplify To simplify an expression you replace it by the simplest or most common symbol having the same value.

Evaluate an expression To evaluate an algebraic expression, or find its value, replace each variable in the expression by a given value and simplify the result.

Power A product of equal factors. The repeated factor is the *base.* A positive *exponent* tells the number of times the base occurs as a factor. Example: $5 \times 5 \times 5 = 5^3$ is a power in which 5 is the base and 3 is the exponent.

Absolute value If x is positive or zero, $|x| = x$.
If x is negative, $|x| = -x$ (reads "the opposite of x").

Symbols

Grouping: ( ) (parentheses) [ ] (brackets) —— (fraction bar)

CAUTION Remember to use the correct order of operations. Expressions inside grouping symbols are simplified first, and then powers. Next, multiplication and division are done *in order from left to right.* Finally, addition and subtraction are done in order from left to right.

Example 1 Use one of the symbols <, =, or > to make a true statement.

a. $5^2 + 3^2 \ \underline{?}\ (5 + 3)^2$

b. $(7 + 2) + 8 \ \underline{?}\ 7 + (2 + 8)$

Solution Find the value of each side. Then compare the results.

a. $5^2 + 3^2 = 25 + 9 = 34$
and $(5 + 3)^2 = 8^2 = 64$
Since 34 is less than 64,
$5^2 + 3^2 < (5 + 3)^2$.

b. $(7 + 2) + 8 = 9 + 8 = 17$
and $7 + (2 + 8) = 7 + 10 = 17$,
so $(7 + 2) + 8 = 7 + (2 + 8)$.

Use one of the symbols <, =, or > to make a true statement.

1. $1 \cdot 4 \ \underline{?}\ 1 \div 4$
2. $4 \cdot 1 \ \underline{?}\ 4 \div 1$
3. $4^2 \cdot 5^2 \ \underline{?}\ (4 \cdot 5)^2$
4. $4^2 + 5^2 \ \underline{?}\ (4 + 5)^2$
5. $\frac{5 + 3}{5 - 3} \ \underline{?}\ \frac{7 + 5}{7 - 5}$
6. $\frac{5 + 4}{5 - 4} \ \underline{?}\ \frac{8 + 4}{8 - 4}$
7. $(8 + 5) + 1 \ \underline{?}\ 8 + (5 + 1)$
8. $(8 - 5) - 1 \ \underline{?}\ 8 - (5 - 1)$
9. $(8 \cdot 5) \cdot 2 \ \underline{?}\ 8 \cdot (5 \cdot 2)$
10. $(16 \div 4) \div 2 \ \underline{?}\ 16 \div (4 \div 2)$

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1-4 Sums and Differences

Objective: To review the rules for adding and subtracting real numbers.

Vocabulary

Rules for addition

1. When adding two numbers with the *same sign*, you add the absolute values of the numbers and keep the sign.
   Example: $-6 + (-3) = -(|-6| + |-3|) = -(6 + 3) = -9$
2. When adding two numbers with *opposite signs*, you subtract the *lesser* absolute value from the *greater* absolute value and keep the sign of the greater absolute value. Example: $4 + -9 = -(|-9| - |4|) = -(9 - 4) = -5$

Rule for subtraction To subtract a number, add its opposite: $a - b = a + (-b)$.
Examples: $4 - (-7) = 4 + 7 = 11$ $\quad -6 - 11 = -6 + (-11) = -17$

Distributive property (of multiplication over subtraction)
Example: $4(5 - 2) = 4 \cdot 5 - 4 \cdot 2 = 12$

Similar terms (or like terms) Terms with the same variables and exponents.
Examples: $5xy^2$ and $9xy^2$ are similar terms, but $6ab$ and $4ab^2$ are not.

Example 1 Simplify: **a.** $-13 + (-40)$ **b.** $-3.4 + 7.2$ **c.** $-14 - 28$

Solution
a. Use the *same sign* rule for addition. The answer will be negative.
$-13 + (-40) = -(|-13| + |-40|) = -(13 + 40) = -53$
b. Use the *opposite signs* rule for addition. The answer will be positive since 7.2 has the greater absolute value.
$-3.4 + 7.2 = |7.2| - |-3.4| = 7.2 - 3.4 = 3.8$
c. Use the rule for subtraction; add the opposite of 28.
$-14 - 28 = -14 + (-28) = -(14 + 28) = -42$

Simplify.

1. $-52 + 17$
2. $-27 - 14$
3. $12 - (-33)$
4. $-16 + (-36)$
5. $-96 - (-28)$
6. $0 - (-23.1)$
7. $-22.7 - (-22.7)$
8. $-16.5 - 12.5$

Example 2 Simplify: **a.** $-17 + 15 - 19 + 31$ **b.** $(3 - 5) - (7 - 2)$

Solution
a. *Method 1:* Add left to right.
$$\underbrace{-17 + 15}_{} - 19 + 31$$
$$\underbrace{-2 - 19}_{} + 31$$
$$\underbrace{-21 + 31}_{}$$
$$10$$

Method 2: Group the negative terms and the positive terms.
$$-17 + 15 - 19 + 31$$
$$\underbrace{(-17 - 19)}_{} + \underbrace{(15 + 31)}_{}$$
$$\underbrace{-36 + 46}_{}$$
$$10$$

b. Be sure to perform the operations inside parentheses first.
$(3 - 5) - (7 - 2) = -2 - 5 = -2 + (-5) = -7$

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1–5 Products

Objective: To review the rules for multiplying real numbers.

Vocabulary

Rules for multiplication

1. The product of two real numbers with *like signs* is a *positive* real number. Examples: $(3)(8) = 24$ $(-6)(-9) = 54$
2. The product of two reals with *opposite signs* is a *negative* real number. Example: $(5)(-14) = -70$
3. A product of nonzero numbers is *positive* if the number of negative factors is *even*. Example: $(-8)(-3)(-5)(-4) = 480$ (4 negative factors)
4. A product of nonzero numbers is *negative* if the number of negative factors is *odd*. Example: $(-6)(5)(-1)(-9) = -270$ (3 negative factors)
5. The absolute value of the product of two or more numbers is the product of their absolute values. Example: $|(3)(-8)(4)| = |3| \cdot |-8| \cdot |4| = 96$

Multiplicative property of 0 The product of any number and zero is zero.

Multiplicative property of −1 The product of any number and negative one is the opposite of that number. Examples: $(5)(-1) = -5$ $(-1)(-18) = 18$

Property of the opposite of a product For all real numbers a and b, $-ab = (-a)(b) = (a)(-b)$. Example: $-(3)(5) = (-3)(5) = (3)(-5) = -15$

Property of the opposite of a sum For all real numbers a and b, $-(a + b) = (-a) + (-b)$. Example: $-[3 + (-19)] = -3 + 19 = 16$

Example 1 Simplify.

a. $\left(\frac{1}{2}\right)(-8)(-6)\left(-\frac{1}{6}\right)$ b. $(4x)(-3y)(2)$ c. $(5 - 7)(-8 + 3)$

Solution

a. Multiply, beginning with the reciprocals.

$$\left(\frac{1}{2}\right)(-8)(-6)\left(-\frac{1}{6}\right) = \left(\frac{1}{2}\right)(-8)(1) = (-4)(1) = -4$$

b. Reorder and regroup factors.

$$(4x)(-3y)(2) = 4(-3)(2)xy = -24xy$$

c. Simplify expressions in parentheses first.

$$(5 - 7)(-8 + 3) = (-2)(-5) = 10$$

Simplify.

1. $4(-2)(-3)(-5)$
2. $(1.4)(-3)(-0.2)$
3. $(-0.6)(-4)(-3)(-5.2)$
4. $\frac{1}{2}(-6)\left(-\frac{1}{12}\right)(-12)$
5. $(4)\left(-\frac{3}{8}\right)(12)$
6. $\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)(0)(-2)(-3)$
7. $5(2x)(-3y)$
8. $\left(-\frac{1}{3}\right)(3u)(-v)$
9. $(4x)(-2y)(-3z)$
10. $(-a)(-b)(-c)$
11. $(4 - 5)(3 + 8)$
12. $(-12 - 3)(2 + 5)$

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1–6 Quotients

Objective: To review rules for dividing real numbers.

Vocabulary

Division To divide by any *nonzero* real number, multiply by its reciprocal.

Examples: $15 \div 3 = 15 \cdot \frac{1}{3} = 5$ $\quad 12 \div \left(-\frac{4}{3}\right) = 12 \cdot \left(-\frac{3}{4}\right) = -9$

Since zero has no reciprocal, *division by zero is undefined.*

Rules for division

1. The quotient of two real numbers with *like signs* is a *positive* real number.
   Examples: $18 \div 3 = 6$ $\quad (-22) \div (-2) = 11$
2. The quotient of two real numbers with *opposite signs* is a *negative* real number.
   Example: $6 \div (-3) = -2$
3. For all *nonzero* real numbers a, b, and c:
   $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ and $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$.

Example 1 Simplify $-6 \div 9 \div \frac{1}{3}$.

Solution Work from left to right.

$$-6 \div 9 \div \frac{1}{3} = \left(-6 \cdot \frac{1}{9}\right) \div \frac{1}{3}$$
$$= -\frac{2}{3} \div \frac{1}{3}$$
$$= -\frac{2}{3} \cdot 3 = -2$$

Simplify.

1. $-63 \div (-9)$
2. $-4 \div 16$
3. $-48 \div 8 \div (-2)$
4. $-18 \div \frac{2}{3}$
5. $-\frac{1}{3} \div \left(-\frac{1}{6}\right) \div (-4)$
6. $(-2)^3 \div [4(-6)]$

Example 2 Simplify: **a.** $\frac{(-6)(-8) \div (-2)}{4(-3)}$ $\quad$ **b.** $\frac{3\left(\frac{3}{4} - \frac{1}{4}\right)}{-\frac{1}{2} \div \frac{3}{2}}$

Solution Simplify the numerator and denominator separately. Then divide.

a. $\frac{(-6)(-8) \div (-2)}{4(-3)} = \frac{48 \div (-2)}{-12} = \frac{-24}{-12} = 2$

b. $\frac{3\left(\frac{3}{4} - \frac{1}{4}\right)}{-\frac{1}{2} \div \frac{3}{2}} = \frac{3\left(\frac{2}{4}\right)}{-\frac{1}{2} \cdot \frac{2}{3}} = \frac{\frac{3}{2}}{-\frac{1}{3}} = \frac{3}{2} \cdot (-3) = -\frac{9}{2}$

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1–7 Solving Equations in One Variable

Objective: To solve certain equations in one variable.

Vocabulary

Open sentence An equation or inequality that contains one or more variables.

Solution set The set of all values of a variable that make an open sentence true. A solution is also called a *root*. A solution is said to *satisfy* an equation.

Transformations Changes that produce equivalent equations (ones with the same solution set). They include:

1. Simplifying either side of an equation.
2. Adding the same number to each side of an equation, or subtracting the same number from each side of an equation.
3. Multiplying (or dividing) each side of an equation by the same *nonzero* number.

Solve an equation Transform the equation into a simpler equivalent one whose solution set is easily seen.

Identity An equation that is satisfied by all values of the variable. The solution set of an identity is the set of all real numbers.

Symbols

$\stackrel{?}{=}$ (Are they equal?) $\quad\therefore$ (therefore)

$\neq$ (is not equal to) $\quad\emptyset$ (empty or null set, the set with no members)

Example 1 Solve $3(2x - 3) = 4x + 7$. (The goal is to get x alone on one side.)

Solution

$6x - 9 = 4x + 7$ — Simplify the left side.

$6x - 9 + 9 = 4x + 7 + 9$ — Add 9 to each side.

$6x = 4x + 16$

$6x - 4x = 4x + 16 - 4x$ — Subtract $4x$ from each side.

$2x = 16$

$\frac{2x}{2} = \frac{16}{2}$ — Divide each side by 2.

$x = 8$

Check: $3[2(8) - 3] \stackrel{?}{=} 4(8) + 7$ — Substitute 8 for x in the *given* equation.

$3(16 - 3) \stackrel{?}{=} 32 + 7$ — Simplify each side.

$3(13) \stackrel{?}{=} 39$

$39 = 39$ ✓

$\therefore$ the solution set is $\{8\}$.

Solve. Check your work.

1. $4x - 6 = 2$
2. $6 = 3x + 3$
3. $\frac{1}{2}x - 4 = -2$
4. $7 - \frac{1}{5}y = -2$
5. $48 - 6x = 2x$
6. $x + 2 = 3x - 6$
7. $4(x - 3) = 2x - 6$
8. $3(1 - y) = 3y$

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Solving Problems; Equations Having the Variable in Both Sides (For use after Section 3–5)

Select each answer from the choices in parentheses. Write the answer in the blank.

1. An equation that is true for every value of the variable(s) is called an ______________. *(identity, impossibility)*

2. If x is an integer, then the next two consecutive integers are ______________. *($2x$ and $3x$, $x + 1$ and $x + 2$)*

Solve. If an equation is an identity or if it has no solution, state that fact.

3. $6w + 8w = 15 + 9w$ ______________
4. $5 + 3d = 13 - 5d$ ______________
5. $6x + 28 = 8x$ ______________
6. $9 = 6a + 21$ ______________
7. $110 + 11y = 0$ ______________
8. $4(5 + 2x) = 4x$ ______________
9. $-5 + 2(x + 4) = 3 + 2x$ ______________
10. $2f - 6f = 8 - 4f$ ______________
11. $5 + 2(x - 4) = 4 + 2x$ ______________
12. $10(2 + x) = -5(2x - 4)$ ______________
13. $28 + 6z = 16$ ______________
14. $5 + |t| = 3$ ______________
15. $2y + 7 = 3(y + 6)$ ______________
16. $4s + 51 + 6s = 17 + 8s$ ______________
17. $0.03(x - 15) = 0.6$ ______________
18. $\frac{2}{3}x + 4 = 4 + \frac{2}{3}x$ ______________

Use an equation to solve each problem.

19. Two integers differ by 12. Three times the smaller one is 11 less than twice the larger one. What are the numbers? ______________

20. The perimeter of a rectangle is 56 and its length is 8 more than its width. Find the length and width. ______________

21. The sum of three consecutive integers is 13 less than four times the smallest. What are the integers? ______________

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2–2 Solving Combined Inequalities

Objective: To solve combined inequalities.

Vocabulary

Conjunction A sentence formed by joining two sentences with the word *and*.

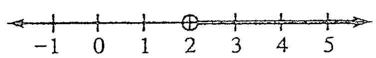
Disjunction A sentence formed by joining two sentences with the word *or*.

Symbols $\geq$ (is greater than or equal to) $a < x < b$ (means "$x > a$ *and* $x < b$")
$\leq$ (is less than or equal to)

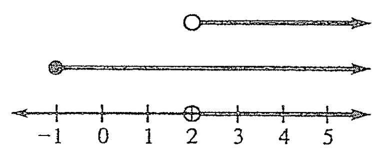
CAUTION A conjunction is true only when *both* sentences are true. A disjunction is true when *either* sentence is true, or when both sentences are true.

Example 1 Graph the solution set of the conjunction $x \geq -1$ *and* $x > 2$.

Solution 1 First find the values of x for which *both* sentences are true. The conjunction is only true when x is greater than 2. To graph, put an open circle at 2 to show that 2 is *not* included in the solution set. Shade to the right of 2.

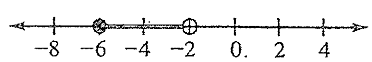


Solution 2 Begin by graphing each inequality separately, above a number line. Then make a graph of the solution set *on* the number line, including only those points that appear in *both* parts.



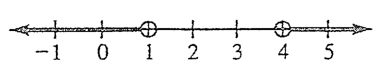
Example 2 Graph the solution set of the conjunction $x \geq -6$ *and* $x < -2$.

Solution Rewrite the conjunction as $-6 \leq x < -2$. Then draw the graph. Or, as an alternative, use the method shown in Solution 2 above.



Example 3 Graph the solution set of the disjunction $x < 1$ *or* $x > 4$.

Solution Find the values of x for which *at least one* of the sentences is true. The disjunction is true for all values of x either less than 1 or greater than 4.



Solve each conjunction or disjunction and graph each solution set that is not empty.

1. $x > 3$ and $x < 7$
2. $x < 5$ and $x < 6$
3. $x \leq -2$ or $x > 2$
4. $2 \geq x > -1$
5. $x > 2$ or $x < -1$
6. $x < 3$ or $x > 3$
7. $x > 4$ and $x \leq -1$
8. $x < 2$ or $x > -2$